Viscous Stability of Compressible Axisymmetric Jets

Philip J. Morris*
The Pennsylvania State University, University Park, Pennsylvania

Abstract

CALCULATIONS of the viscous, spatial stability characteristics of a compressible axisymmetric jet are presented. The asymptotic solutions to the stability equations in regions of constant mean flow properties are developed and used as the boundary conditions for a numerical integration of the compressible stability equations in cylindrical coordinates. Increasing the Mach number is found to stabilize the flow, as is decreasing the Reynolds number. The helical mode is found to be more unstable than the axisymmetric mode for higher Mach numbers.

Contents

The recognition that turbulent flows possess large-scale structures that are responsible for the enhanced mixing properties of turbulent flows has been stimulated by the experiments of Brown and Roshko.[1] In the case of free shear flows such as mixing layers, jets, and wakes, the large-scale motions have measured characteristics which appear related to the hydrodynamic stability properties of the mean flow profile. The modeling of these large-scale motions as a random superposition of instability waves or normal modes has proved successful in describing the statistical properties of turbulence in a two-dimensional mixing layer[2] and an axisymmetric jet.[3] Another observed phenomenon which appears to be associated with the presence of excited instability waves or large-scale structures is that of broadband jet noise amplification.[4,5] It has been observed that when a jet is excited by a pure tone upstream of the jet exit both the jet turbulence and the noise radiation at all frequencies are modified. This phenomenon may be modeled by assuming that the upstream excitation stimulates instability waves in the jet. If a detailed analysis of the large structures and other components of the turbulent flow is required, a knowledge of the instability wave properties at all radial locations is needed. In this case, a viscous form of stability analysis is necessary if decaying instability waves are to be included. Since Moore's[5] measurements also indicated a dependence of the jet's sensitivity on the jet Mach number, compressibility effects should be included in the stability analysis. In the present investigation the viscous incompressible stability analysis of Morris[6] is extended to include the effects of compressibility.

The linear stability equations solved in the present investigation are those developed by Dunn and Lin,[7] in which the viscous effects in the energy equation are neglected and the viscous terms in the momentum equations are characterized by their incompressible form. This restricts the analysis to moderate Mach numbers and flows with little variation in the mean static temperature. If the mean velocity and temperature are taken to be functions of radial position alone, the locally parallel flow approximation, then the disturbance equations have a separable form with wave-like variation in the axial direction and time and a periodic variation in the azimuthal direction determined by an azimuthal mode number. The disturbance equations then reduce to a system of four coupled, ordinary differential equations. The boundary conditions on the axis are kinematic in origin and the disturbances are required to decay outside the jet flow.

The key to the numerical solution of these equations depends on finding their analytic solution in regions of constant mean properties: in the potential core or outside the jet flow. These solutions have been obtained using the methods described by Morris.[6] The equations are solved using a series expansion for small radius. The form of series solution is then identified with the series expansion for the modified Bessel functions of the first kind. The solutions outside the jet flow may then be inferred to be in the form of Hankel functions of the first kind with the identical arguments. The asymptotic solutions are found to be of the form:

$n=0$

$$\hat{p}=A_1 H_0^{(1)}(i\lambda^* r) \tag{1}$$

$$\hat{u}=A_1\frac{\alpha(Re-iM^2\Omega_0)}{\rho_0\Omega_0 Re}H_0^{(1)}(i\lambda^* r)+A_3 H_0^{(1)}(i\lambda r) \tag{2}$$

$$\hat{v}=-A_1 i\lambda^*\frac{Re-iM^2\Omega_0}{\rho_0\Omega_0 Re}H_1^{(1)}(i\lambda^* r)-A_3\frac{i\alpha}{\lambda}H_1^{(1)}(i\lambda r) \tag{3}$$

$n\neq 0$

$$\hat{p}=A_1\frac{\rho_0\Omega_0 Re}{\alpha(Re-iM^2\Omega_0)}H_n^{(1)}(i\lambda^* r) \tag{4}$$

$$\hat{u}=A_1 H_n^{(1)}(i\lambda^* r)+A_3 H_n^{(1)}(i\lambda r) \tag{5}$$

$$\hat{v}=-A_1\frac{i}{\alpha}\frac{\mathrm{d}}{\mathrm{d}r}\{H_n^{(1)}(i\lambda^* r)\}-A_3\frac{\alpha}{\lambda}H_{n+1}^{(1)}(i\lambda r)$$
$$-A_5\frac{in}{r}H_n^{(1)}(i\lambda r) \tag{6}$$

$$\hat{w}=A_1\frac{n}{\alpha r}H_n^{(1)}(i\lambda^* r)+A_3\frac{i\alpha}{\lambda}H_{n+1}^{(1)}(i\lambda r)$$
$$+A_5\frac{\mathrm{d}}{\mathrm{d}r}\{H_n^{(1)}(i\lambda r)\} \tag{7}$$

where

$$\lambda^{*2}=(\alpha^2 Re-i\lambda^2 M^2\Omega_0)/(Re-iM^2\Omega_0) \tag{8}$$

$$\lambda^2=\alpha^2-i\rho_0\Omega_0 Re \tag{9}$$

and

$$\Omega=\omega-\alpha\bar{u} \tag{10}$$

Presented as Paper 80-1004 at the AIAA 6th Aeroacoustics Conference, Hartford, Conn., June 4-6, 1980; submitted June 30, 1980; synoptic received July 2, 1982. Full paper available from AIAA Library, 555 West 57th Street, New York, N.Y. 10019. Price: Microfiche, $4.00; hard copy, $8.00. **Remittance must accompany order.**

*Associate Professor, Department of Aerospace Engineering. Member AIAA.

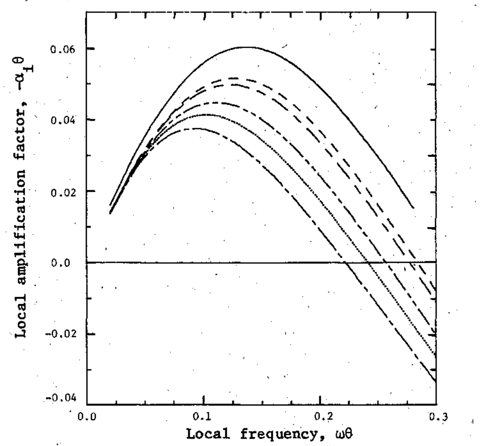

Fig. 1 Effect of Mach number on the variation of amplification factor with frequency. $n=1$, $Re\theta=80.0$. $M=0$, ---; $M=0.4$, —; $M=0.8$, ----; $M=1.0$, · · ·; $M=1.2$, ——. Inviscid solution, $M=0$, —.

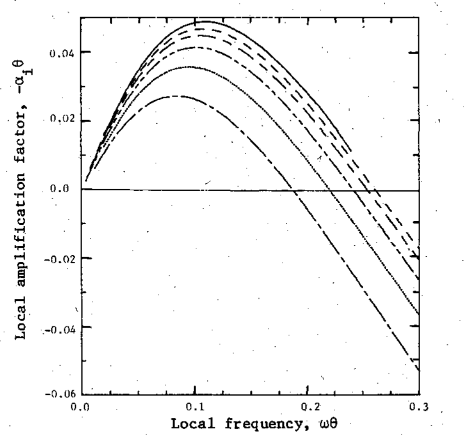

Fig. 2 Effect of Reynolds number on the variation of amplification factor with frequency. $n=1$, $M=1.0$. $Re\theta=20.0$, ——; $Re\theta=40.0$, · · ·; $Re\theta=80.0$, ----; $Re\theta=160.0$, ---; $Re\theta=320.0$, – – –. Inviscid solution, $M=1.0$, —.

$\hat{u}$, $\hat{v}$, $\hat{w}$, and $\hat{p}$ give the radial variation of the velocity fluctuations in the axial, radial, and azimuthal directions and the pressure, respectively. $\bar{u}$ is the axial mean velocity and ρ is the mean density. α is the axial wavenumber, ω is the radian frequency, and n is the azimuthal mode number of the instability wave. Re is the Reynolds number based on jet exit radius and velocity and M is the jet exit Mach number. The subscript 0 denotes a constant mean flow property. The real parts of λ and λ^* are taken to be positive so as to satisfy the outer boundary conditions. Similar forms, in terms of modified Bessel functions may be written for the solutions in the potential core or very close to the jet axis. It should be noted that for $M=0$, $\lambda^*=\alpha$, and the viscous incompressible solutions are recovered. In the inviscid case, for which $1/Re=0$, $\lambda^{*2}=\alpha^2-M^2\Omega_0^2\rho_0$, and the compressible, inviscid solutions are found.

The forms of solution contained in Eqs. (1-7) provide the starting conditions for the numerical integration of the stability equations through a region in which the mean flow properties are varying. This integration has been performed using an orthonormalization procedure to maintain the linear independence of the solutions. The results of this numerical integration may be matched to the corresponding form of the analytic solutions in the potential core region or on the axis of the jet. This matching provides the criterion for determining the eigenvalue.

The mean velocity profile considered in the calculations is representative of jet profiles towards the end of the potential core. The same profile has been used by Michalke[8] and Morris,[6] and is given by

$$\bar{u}=\frac{1}{2}\left\{1+\tanh\left[\frac{1}{4\theta}\left(\frac{1}{r}-r\right)\right]\right\} \tag{11}$$

where θ is the local momentum thickness and its variation was used by Michalke[8] to represent the influence of axisymmetry on the jet stability. The mean density is related to the velocity using a Crocco relationship. Calculations have been performed for $\theta=0.16$ and a jet static temperature ratio of 1.0.

In view of the large number of parameters that enter the problem in compressible viscous analyses, only a limited number of calculations have been performed. The trends observed in the limiting cases of inviscid compressible flow and viscous incompressible flow are also found in the present results. The variation of the local growth rate for the helical, $n=1$, mode is shown in Fig. 1 for a local Reynolds number, $Re\theta=80$. As the Mach number increases, so the maximum growth rate decreases and the range of amplifying frequencies decreases. Growth rates for the $n=0$ mode are found to decrease more rapidly than the helical mode. This indicates that for high Mach numbers the helical mode of instability would be more likely to occur naturally. The variation with Reynolds number of the local growth rate for the helical mode is shown in Fig. 2 for a Mach number of 1.0. The inviscid result at this Mach number is also shown and agrees with Michalke's[8] result. The trends are the same as those calculated by Morris[6] for the incompressible case. Increasing the Reynolds number increases the growth rates at all frequencies.

A number of simplifying assumptions have been made in the development of the compressible stability equations. Extension of the method presented here to a system of equations which includes such effects as temperature dependent molecular properties appears straightforward. However, if the Reynolds number is based on an eddy viscosity, which provides the most useful application of the present analysis, the present system of equations should be adequate.

Acknowledgments

This work was supported by the NASA Langley Research Center under Grant NSG 1580. The technical monitor is Dr. T.B. Gatski.

References

[1]Brown, G. and Roshko, A., "On Density Effects and Large Structure in Turbulent Mixing Layers," *Journal of Fluid Dynamics*, Vol. 64, Pt. 4, 1974, pp. 775-816.

[2]Tam, C.K.W. and Chen, K. C., "A Statistical Model of Turbulence in Two-Dimensional Mixing Layers," *Journal of Fluid Mechanics*, Vol. 92, Pt. 2, 1979, pp. 303-326.

[3]Plaschko, P., "Stochastic Model Theory for Coherent Turbulent Structures in Circular Jets," *Physics of Fluids*, Vol. 24, No. 2, 1981, pp. 187-193.

[4]Bechert, D. and Pfizenmaier, E., "On the Amplification of Broadband Jet Noise by a Pure Tone Excitation," *Journal of Sound and Vibration*, Vol. 43, Pt. 3, 1975, pp. 581-587.

[5]Moore, C. J., "The Role of Shear Layer Instability Waves in Jet Noise," *Journal of Fluid Mechanics*, Vol. 80, 1977, pp. 321-367.

[6]Morris, P. J., "The Spatial Viscous Instability of Axisymmetric Jets," *Journal of Fluid Mechanics*, Vol. 77, Pt. 3, 1976, pp. 511-529.

[7]Dunn, D. W. and Lin, C. C., "On the Stability of the Laminar Boundary Layer in a Compressible Fluid," *Journal of the Aeronautical Sciences*, Vol. 22, No. 7, 1955, pp. 455-477.

[8]Michalke, A., "Instabilität eines kompressiblen runden Freistrahls unter Berucksichtigung des Einflusses der Strahlgrenzschichtdicke," *Zeitschrift fuer Flugwissenschaften*, Vol. 19, 1971, pp. 319-328.